2024-2025
Fall Semester

Course of Power System Analysis

The per-unit method

Prof. Mario Paolone
Distributed Electrical Systems Laboratory
École Polytechnique Fédérale de Lausanne (Switzerland)

Outline

Per-unit quantities

Single phase systems

Three phase systems

Per-unit quantities

Electrical quantities such as voltage, current, power or impedance are often expressed as a percentage or **per unit of a base or reference** value specified for each. For instance, if a base voltage of 120 kV is chosen, voltages of 108, 120, and 126 kV become 0.90, 1.00, and 1.05 per-unit.
The per-unit value of any quantity is defined as the ratio of the quantity to its base.

If the given quantity is expressed as a complex number $\bar{A} = A_1 + jA_2$, **the same base value is assumed for both the real part and the imaginary coefficient:**

$$\bar{a} = \frac{\bar{A}}{A_b} = \frac{A_1 + jA_2}{A_b} = \frac{A_1}{A_b} + j\frac{A_2}{A_b} = a_1 + ja_2$$

As a direct consequence, the **module of $\bar{A}$ divided by the base A_b is equal to the module of $\bar{a}$** as shown in the following equation:

$$\frac{|\bar{A}|}{A_b} = \frac{\sqrt{A_1^2 + A_2^2}}{A_b} = \sqrt{\frac{A_1^2}{A_b} + \frac{A_2^2}{A_b}} = \sqrt{a_1^2 + a_2^2} = |\bar{a}|$$

Per-unit quantities

The choice of the bases has to be **consistent.**
A system of base values is consistent if the per-unit value of a quantity, which depends on other quantities according to a "**physical law**", can be obtained from the per-unit values of these quantities using the above-mentioned "physical law".

In an electrical system the bases for voltages (V_b), **currents** (I_b), **powers** (A_b) **and impedances** (Z_b) **are so related that the selection of any two of them determines the base values of the remaining two.**

Example:
If we specify the base values of current and voltage, base impedance and base kVA can be determined as follow:

- The **base kVA** in single-phase systems is the product of base voltage in kilovolts and base current in amperes.
- The **base impedance** is that impedance which will have a voltage drop across it equal to the base voltage when the current flowing in the impedance is equal to the base value of the current

Outline

Per-unit quantities

Single phase systems

Three phase systems

Single phase systems

Given a single-phase network, the **base value** E_b **for the voltages** and the **base value** A_b **for the powers** are chosen.
Consequently, the base values for currents and impedances (or admittances) are:

$$I_b = \frac{A_b}{E_b}$$

$$Z_b = \frac{E_b}{I_b} = \frac{E_b^2}{A_b}, \left(Y_b = Z_b^{-1} = \frac{A_b}{E_b^2} \right)$$

Once the base values are defined it is possible to express the system in per unit as follow:

$$e = \frac{E}{E_b}, i = \frac{I}{I_b}, a = \frac{A}{A_b}$$

where:

$$p = \frac{P}{A_b}, q = \frac{Q}{A_b}$$

Single phase systems

If Z is the absolute value of the impedance of an element of the network, the per-unit of this impedance z is:

$$z = Z\frac{1}{Z_b}$$

and therefore, by recalling the definition of $Z_b = \frac{E_b}{I_b} = \frac{E_b^2}{A_b}$, we get:

$$z = Z\frac{I_b}{E_b} = Z\frac{A_b}{E_b^2}$$

The impedance expressed in per-unit is therefore defined by the ratio between the voltage drop determined by the impedance z when the base current I_b pass through and the base voltage E_b.

Example: if the impedance of a network element, crossed by the base current, causes a voltage drop equal to 10% of the base voltage, it will be $z\ =\ 0.1\,pu$.

Single phase systems

Sometimes the per-unit impedance of a component of a system is expressed on a base other than the one selected as base for the part of the system in which the component is located.

Since **all impedances in any one part of a system must be expressed on the same impedance base** when making computations, it is necessary to have a means of converting per-unit impedances from one base to another.

By recalling

$$z = Z\frac{A_b}{E_b^2}$$

We can observe that **per-unit impedance is directly proportional to the base power A_b** and **inversely proportional to the square of the base voltage E_b**. Therefore, to change from per-unit impedance on a given base to per-unit impedance (1) on a new base (2), the following equation applies:

$$z_2 = z_1\left(\frac{E_{b_1}}{E_{b_2}}\right)^2\left(\frac{A_{b_2}}{A_{b_1}}\right)$$

Outline

Per-unit quantities

Single phase systems

Three phase systems

Three-phase systems

- All the previous dependencies also apply in three-phase systems if we consider **the per-phase power as the base value** $A_{b_{1\phi}}$ and **the base value of the line-to-ground voltages as** E_b.

- **However, information about three-phase systems is generally provided referring to the three-phase power and the line-to-line voltage.**

- As a result, it might be easier for three-phase systems to choose the **base value** V_b **for the line-to-line voltage** and **the base value** A_b **for the three-phase power**.

- The **coherent values** of the base current I_b and impedance Z_b are therefore determined as follows:

$$I_b = \frac{A_{b_{1\phi}}}{E_b} = \frac{A_b}{3} \cdot \frac{\sqrt{3}}{V_b} = \frac{A_b}{\sqrt{3}V_b}$$

$$Z_b = \frac{E_b^2}{A_{b_{1\phi}}} = \left(\frac{V_b}{\sqrt{3}}\right)^2 \cdot \frac{3}{A_b} = \frac{V_b^2}{A_b}$$

Three-phase systems

- **Balanced and symmetrical three-phase circuits** can be solved by **referring to the corresponding single-phase equivalent circuits**.
- Although a line-to-line voltage may be specified as base, the voltage in the single-phase circuit required for the solution is still the line-to-ground voltage.
- The base line-to-ground voltage E_b is the base line-to-line voltage V_b divided by $\sqrt{3}$.
- Since this is also the ratio between line-to-line and line-to-ground voltages of a balanced three-phase system, **the per-unit value of a line-to-neutral voltage on the line-to-ground voltage base is equal to the per-unit value of the line-to-line voltage on the line-to-line voltage base if the system is balanced.**

$$v = \frac{V}{V_b} = \frac{\sqrt{3}E}{V_b} = \frac{E}{E_b} = e$$

Therefore, the voltages acting in the equivalent single-phase circuit, with impedances and admittances expressed in pu., represent indifferently the line-to-ground or the line-to-line voltage.
By applying the line-to-line voltage, we obtain that the powers flowing in the equivalent single-phase circuit are three-phase powers in pu.

Per-unit impedances

- Manufacturers usually specify the impedance of a piece of apparatus in per-unit on the base of the **nameplate rating**.

- The per-unit impedances of machines of the same type and widely different rating usually lie within a narrow range. When the impedance is not known definitely, it is generally possible to select from tabulated average values a per-unit impedance which will be reasonably correct.

- Experience in working with per-unit values brings familiarity with the proper values of per-unit impedance for different types of apparatus.

- **The per-unit impedance, once expressed on the proper base, is the same referred to either side of any transformer (to be seen in the lecture on transformers)**

- **The way in which transformers are connected in three-phase circuits does not affect the per-unit impedances of the equivalent circuit** although the transformer connection does determine the relation between the voltage bases on the two sides of the transformer **(to be seen in the lecture on transformers).**